ME-412 : DIC analysis of a crack tip stress fields on a polyacrylamide hydrogel

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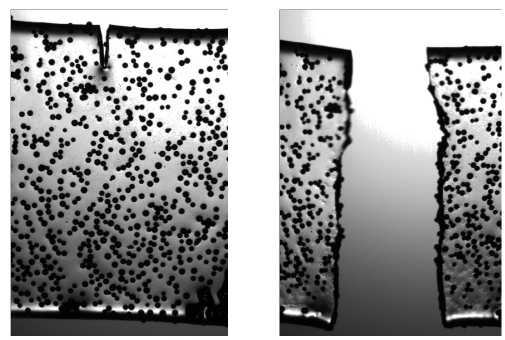
The present study experimentally investigated the Linear Elastic Fracture Mechanics (LEFM) predictions over a polyacrylamide hydrogel. More precisely, the aim is to measure the critical stress intensity factor K_c of an hydrogel with a specific chemistry. This factor is used to predict when a crack will propagate and then cause failure in the material. In the LEFM theory, K_I of a Mode I fracture is related to the displacements at each point around the crack tip. These displacements are computed using a digital image correlation (DIC) analysis during a tensile test. Then, the displacements are compared with the theoretical ones obtained with the LEFM theory. This theory is based on elastic materials, but the hydrogel used is assumed to by hyperelastic.

Linear Elastic Fracture Mechanics (LEFM) is a branch of fracture mechanics which deals with the study of precracked materials [1]. The assumption of this theory is that the material follows the Hooke's law, thus behaves elastically, up to the point of fracture. LEFM is based on the stress intensity factor K which describes the severity of a crack in a specific material. The critical stress intensity factor K_c also plays a role in this theory. It describes the toughness of the material. When $K \geq K_c$, the crack will propagates, leading to the fracture of the material. K_c is a material property and can be determined.

The material used in this paper is a polyacrylamide hydrogel. Hydrogel is a material composed principally by water. Each chemistry can be described with three specific parameters [2] : water-to-monomer molar ratio W, crosslinker-to-monomer molar ratio C and polymer-to-hydrogel weight ratio Φ. Hydrogels are incompressible material [3] and follow the neo-Hookean model for incompressible materials, thus they are hyperelastic materials. The paper aims at studying the fracture mechanics of hydrogel and at answering this question : can the LEFM model be applicable on hydrogel even though it is an hyperelastic material that in this study is characterized by an incompressible neo-Hookean model ?

In order to answer this question, a Digital Image Correlation (DIC) analysis was performed on images taken during a tensile test of precracked hydrogel samples. DIC is an optical method used to measure displacements and strains of objects under a specific loading. In this paper, the object is a polyacrylamide hydrogel with a crosslinker-to-monomer ratio $C = 4.33\%$ and a water-to-monomer ratio $\Phi = 13.8\%$. The experiment consists of a tensile test until fracture of a precracked sample. Before the test, a pattern needs to be applied on the sample as shown in Figure 1a. This pattern is determinant for the DIC analysis as discussed later in this paper. The pattern used in this paper was made with glass beads disposed on the sample. This method minimizes the effect of the surface tension induced by the water. Then the patterned sample is precracked and stretched. To obtain images for the DIC analysis, the loading is recorded by a 2MPx camera (1920 x 1080 pixels). The setup needs to be stable to obtain the less noisy images possible.



(a) Before loading (b) After loading

FIG. 1: Example of pattern before and after the loading on a hydrogel sample. (a) The sample is precracked before the loading in order to do a fracture test. (b) After the loading, the sample is broken and the pattern does not seem affected.

When the images acquisition is done, the goal is to analyze the data using DIC. The DIC was performed with the *Matlab* toolbox : *Ncorr* [4]. This open-source software is able to compute displacements and strains for the dataset by applying the fundamental laws of continuum mechanics. DIC stands for digital image correlation. It starts with a reference image on which the coordinates of a subset of points is taken. Then, for each image, it search for the best transformation vector to fit to the displaced subset, working by minimizing the correlation. Once displacements and strains are computed for a specific sample, the stress field was computed and visualized as in Figure 6.

The testing was performed over four different samples. As said above, all samples have the same crosslinker-to-monomer ratio $C = 4.33\%$ and same polymer-to-hydrogel ratio $\Phi = 13.8\%$, these material properties are intrinsic to the samples tested. Dimensions of samples are reported in Table I as initial gap length of the tensile test (measured on the test site), the initial crack length (measured on the computer with an image of reference) and the width (measured on the test site). The thickness has been measured for one sample and is assumed constant for all samples: $t = 5.5 \times 10^{-4}$ [m]

N°	l_i [mm]	l_c [mm]	w [mm]
1	12.01	1.18	11.93
2	11.85	0.978	11.44
3	11.31	1.61	10.92
4	11.85	1.66	11.46

TABLE I: Table summarizing the dimensions of the four samples tested in this paper. Dimensions are respectively the initial gap length of the tensile test, the crack length and the width.

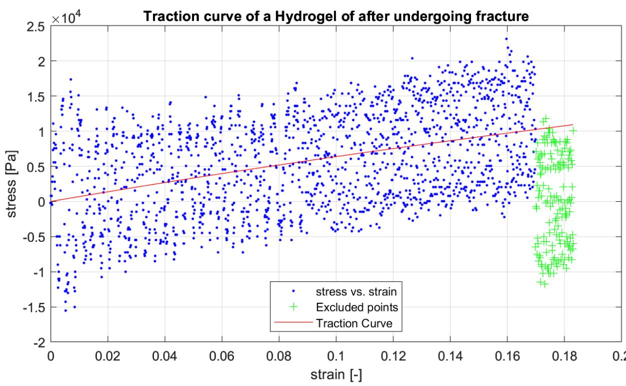


FIG. 2: Traction curve from the data collected during the fracture of the 4th sample. Data fitted with the *Matlab* toolbox *curveFitter*, following the Equation 1 describing the relation between engineering stress and engineering strain for an incompressible neo-Hookean material subjected to uniaxial tension

For incompressible neo-Hookean material under uniaxial tension the relationship between the engineering stress, σ, and engineering strain, ε, is described by the following equation:

$$\sigma = 2C_1 \left((\varepsilon + 1) - \frac{1}{(\varepsilon + 1)^2} \right) \tag{1}$$

where the factor $2C_1$ represents the shear modulus μ [5]. The data extracted from the 4 samples show either inconsistent values or incomplete dataset. Therefore only one value of C1 has been computed, over the dataset given after fracturing the sample 4. After fitting the dataset with the previously explained model, a value of $C_1 = 11670$ [Pa] has been found. Thus giving a shear modulus $\mu = 2 \times C_1 = 23340$ [Pa].

The crack tip displacement fields using a LEFM model for Mode I, opening mode where the tensile stress is normal to the plane of the crack [6], are given by equations 2 and 3

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right) \left[\kappa - 1 + 2\sin^2\left(\frac{\theta}{2}\right)\right] \tag{2}$$

$$u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}} \sin\left(\frac{\theta}{2}\right) \left[\kappa + 1 - 2\cos^2\left(\frac{\theta}{2}\right)\right] \tag{3}$$

where K_I is the stress intensity factor for Mode I and $\kappa = (3-\nu)(1+\nu)$ for plane stress. Hydrogels are considered as incompressible materials so the Poisson's ratio is $\nu = 0.5$. The qualitative results of the crack tip displacement fields using the LEFM model are shown in Figure 3.

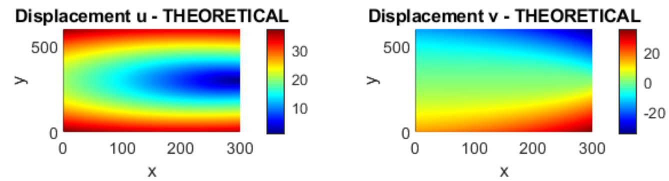


FIG. 3: Visualization of the displacements field predicted by LEFM respectively in the x-direction and in the y-direction obtained by equations 2 and 3 in case of a crack tip on the right

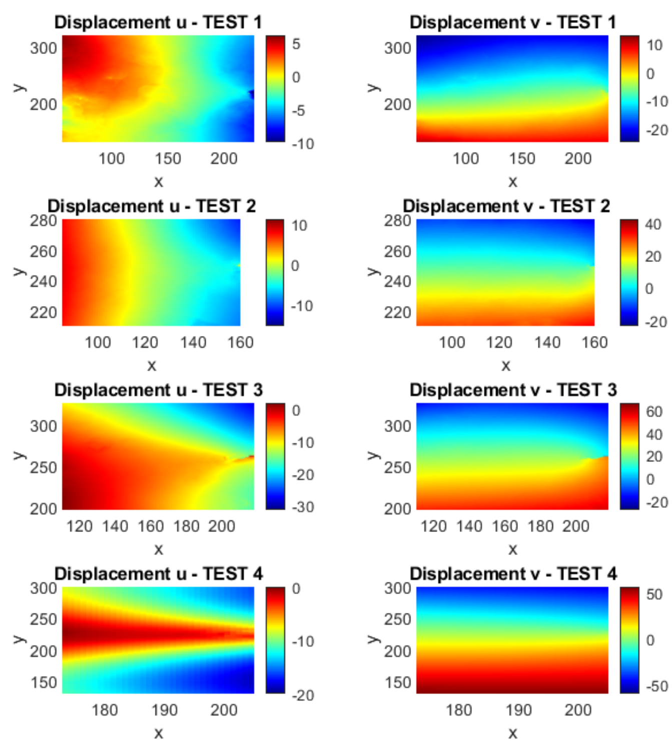


FIG. 4: Visualization of the displacements field obtained after DIC analysis for respectively test 1, 2, 3 and 4 from the top to the bottom. The scales of the colorbar are normalized in case of a crack tip on the right

Using DIC, the experimental displacement fields for the four samples tested were computed and are shown in Figure 4. When compared to the qualitative results of the theoretical displacement fields it shows that the vertical displacement fields (displacement v) are better predicted than those for the horizontal displacement fields (displacement u) where there is little resemblance.

To obtain a quantitative result on the applicability of LEFM over the hydrogel of interest, the J-integral of each sample was computed. The J-integral is a path-independent integral which represents the energy release rate at the moment the crack is propagating. Before computing the J-integral, it is fundamental to define the strain energy W. W represents the work done to deform the material and for a incompressible neo-Hookean material is given by

$$W = \frac{1}{2}\mu(I_1 - 3)$$

where $I_1 = tr\mathbf{C} = tr(\mathbf{F}^T\mathbf{F})$ or the first invariable of the Right Cauchy-Green deformation tensor $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ [7]. Then it is possible to compute the J-integral as following [8]

$$J = \int_C W n_1 ds - t_i \frac{\partial u_i}{\partial x_1} ds \tag{4}$$

where C is a closed path around the crack tip, W is the strain energy, $\mathbf{n}$ is the normal vector, t_i is the stress defined by equation 1 and $\mathbf{u}$ is the displacement vector.

It is possible to link the J-integral with the stress intensity factor K obtained by LEFM [6]. The relation for plane stress is given by

$$J = \frac{K_I^2}{E} \tag{5}$$

where E is the Young's modulus and can be computed using the shear modulus μ and the Poisson's ratio $\nu = 1.5$ for incompressible material with the following relationship

$$E = 2\mu(1 + \nu) = 4C_1(1 + 0.5) = 6C_1 = 70\ kPa$$

The measure of fracture toughness of a material, G_C, can be related to the J integral if computed just before the crack starts to propagate with the following equation:

$$J_C = G_C \tag{6}$$

Using equations 4 and 5, J-integral and K_I values were obtained for the four samples and are reported in the Table II. As the value of the J integral has been computed on the last image before fracture, it is assumed that the stress intensity factor computed is the critical one. The results show similar values for test 3 and 4 since test 1 and 2 are one order of magnitude larger. This can be explained by a non-uniform pattern on the samples leading to miscalculations during the DIC analysis. This issue will be discussed later.

N°	J $[J/m^2]$	K_{IC} $[Pa\sqrt{m}]$
1	426.82	5466.8
2	103.18	2687.9
3	46.44	1803.3
4	44.33	1761.8

TABLE II: Table summarizing for each sample the value of the J-integral for a path around the crack tip and the corresponding stress intensity factor for the Mode I loading K_I.

In order to quantitatively compare the experimental values obtained above, it is important to obtain an theoretical value. This value is basically a function of the material properties. Lake and Thomas defined an equation, using scaling analysis, to obtain a theoretical estimate of the toughness Γ_{th} [9]. This value is a function of the polymer-to-hydrogel $\Phi = 13.8\%$, the crosslinker-to-monomer ratio $C = 4.33\%$, the number of monomers per polymer chain $n = 0.5/C = 1$, the volume per monomer $V = 10^{-28}\ m^3$, the length per monomer $a = 4.6{*}10^{-10}\ m$ and the covalent energy of a C-C bond $J = 5 * 10^{-19}\ J$ [10]. The following equation describes the computation of Γ_{th}

$$\Gamma_{th} = \phi\sqrt{n} aJ/V = 1.0708\ J/m^2$$

Using this value to now compute the theoretical critical stress intensity factor $K_{IC,th}$ using $G_C = \Gamma_{th}$ and equation 5leads to $K_{IC,th} = 273.8\ Pa/\sqrt{m}$. This value is really far away from the one obtained above. This can be explained by multiple factors as explained below.

While comparing the theoretical and experimental results, the values obtained are too different to attest the initial hypothesis. The critical stress intensity factors are significantly different, but the displacement field in the vertical axis matches (on sample 4 particularly). Therefore, a conclusion can be taken: LEFM is not applicable on hyper-elastic materials such as the hydrogels tested here. Despite making those conclusions, it is important to note that some simplifications have been made and errors can occur while testing. Here are some explanations of the potential errors.

The differences between the qualitative results of the theoretical displacement fields and the experimental displacement fields for the four samples might be due to the fact that hydrogels are hyperelastic materials. Therefore, when stretched vertically, the horizontal displacement is much greater than for a linear elastic material. The fracture is of Mode I resulting in much greater vertical displacements than horizontal. The density of the glass particles on the surface of the hydrogels might not have been as dense as needed for the software used for this experience to characterize the displacement field. The hyperelastic behavior of the hydrogel might have been influence by the difference in hydration of the samples during the test. Some samples got dehydrated during the fracture test, leading to different mechanical behavior of the samples tested.

The data collected during the test of fracture is inconsistent, as a matter of fact, the values obtained for the displacement field of the tensile test bench moves at a speed varying with time (see Figure 5). This causes some issue while calculating the coefficient C_1. The theoretical speed has been calculated by multiplying the constant speed $v = 40\frac{\mu m}{s}$. The real displacement has been calculating by adding the position value of axis 1 and axis

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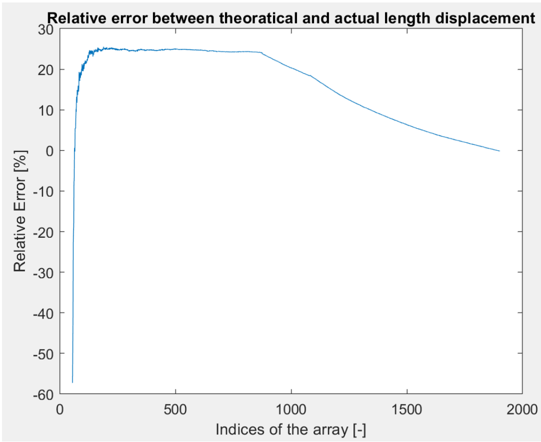

FIG. 5: Relative error for the theoretical and real displacement given by the test machine.

It should be mentioned that the theoretical estimation of the toughness made by Lake and Thomas has not been proved to be fully accurate when predicting the toughness of the material and therefore could result in an inaccurate assumption in this case (based on a discussion with a TA in the course EMEM at EPFL on 22.11.23)

In order to confirm the results presented here, it would be necessary to carry out a larger number of tests on the same material, thus making it possible to reduce the probability of errors that may have been made during the testing phase. Having a larger range of results is ideal for demonstrating their precision and veracity. In the same perspective, finding a more appropriate model than the incompressible neo-Hookean one for data interpolation would also be important to improve the accuracy of the results.

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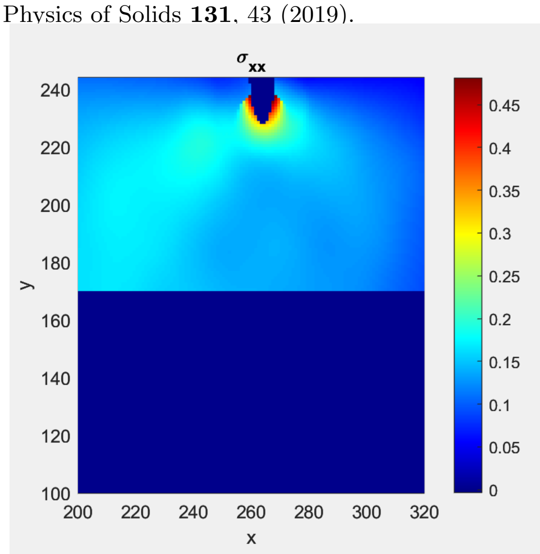

FIG. 6: Example of a stress field in the direction X-X obtained and calculated on *Matlab* with strain field given by the toolbox *Ncorr*.